PAPERS | MARCH 01 2025

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Am. J. Phys. 93, 215–222 (2025)
https://doi.org/10.1119/5.0228366

Kepler's Moon puzzle—A historical context for pinhole imaging

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(Received 12 July 2024; accepted 25 November 2024)

In 16th-century European astronomy, determining the sizes of the Sun and Moon using a pinhole camera was common. However, calculating the Moon's diameter from the concave segment of the partially obscured Sun yielded puzzling results due to a lack of a comprehensive theory of the influence of the aperture on the image. This inconsistency led Tycho Brahe to question prevailing assumptions in celestial mechanics. Recognizing this, Johannes Kepler conducted measurements during a solar eclipse in Graz on July 10, 1600 and soon developed a theory of the pinhole camera that remains valid today. In this article, we recount the historical episode leading to Kepler's theory through original works, complemented by a series of illustrative experiments for classroom use. This historical case study offers a rich context for reflecting on Nature of Science aspects within physics education.
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I. INTRODUCTION

By the late 16th century, pinhole camera projection had become a common method among European astronomers for observing and measuring the angular size of the Sun and eclipses despite the lack of a comprehensive theory.[1,2] However, this method led to deviations from the observations with the naked eye: the solar angles derived from the pinhole images of the Sun always turned out to be too large. Tycho Brahe, the royal astronomer in Prague, was among the first to consider the influence of aperture size on the pinhole image, achieving more accurate results by subtracting the aperture diameter from the Sun's projected image.[3] Another challenge was determining the angular size of the Moon during a solar eclipse (Fig. 1): using the concave edge of the partially obscured Sun resulted in a significantly smaller apparent Moon size as compared with results of direct observation. Brahe's conclusions from this "Moon puzzle" pointed to anomalies in celestial motions, which intrigued the then 28-year-old Johannes Kepler.[4]

In the early afternoon of July 10, 1600, Kepler prepared to observe an upcoming solar eclipse from the marketplace at Graz, Austria.[5] Predictions indicated that the Moon would obscure nearly half the Sun at its peak. Using a homemade pinhole camera instrument, Kepler hoped to determine the apparent size of both the Sun and Moon from their projected images. This endeavor, he believed, would shed light on the inconsistencies noted by Brahe. Around 1:30 PM, Kepler positioned his device and meticulously documented his measurements in his notebook. Unbeknownst to him, he was on the verge of formulating a comprehensive theory of the pinhole camera, which would not only resolve the Moon puzzle but also fundamentally shift the understanding of optics.[6]

The investigation of the Moon puzzle during the solar eclipse of July 1600 is now recognized as a significant landmark in the history of science. Kepler's development of pinhole camera theory established the principles of geometric optics, which are still taught today. The documentary evidence for this episode is remarkably well-preserved. Kepler's notebook entries along with correspondence from Brahe and Kepler as well as subsequent optical works have survived in near-complete form. From an educational standpoint, this allows for the exploration of the development of pinhole camera theory through authentic documents within a historically accurate context. By retracing this process, students can not only learn the geometric rules of pinhole camera theory as technical content but also gain insight into key aspects of the nature of the development of science through a historical case study. In this article, we explore the Moon puzzle from historical and experimental perspectives.

The outline of this paper is as follows. In Sec. II, we trace the origins of the Moon puzzle from ancient times through Brahe's conclusions to Kepler's resolution inspired by the solar eclipse of July 10, 1600. Section III presents a series of classroom-appropriate observations and experiments motivated by the development of the pinhole camera. These activities are intended to emphasize some methodological and scientific aspects of the historical investigation while familiarizing students with experimental aspects of the Moon puzzle by means of practical experience.

Kepler's resolution of the Moon puzzle culminated in his comprehensive theory of "light figures" as detailed in his seminal 1604 work *Ad Vitellionem Paralipomena*. In a follow-up paper, we will retrace theoretically, experimentally, and mathematically how Kepler generalized his concept of pinhole camera theory to the formation of shadow images ("light figures").

II. HISTORY OF THE MOON PUZZLE

A. The pinhole camera as an astronomical instrument

The study of image formation through apertures has fascinated scholars since ancient times. At the heart of the issue was primarily the "window problem" or the "problem of Sun coins," specifically the question of how pinhole images of the Sun are formed by apertures of finite size. In the pseudo-aristotelian "problemata physica" (4th century B.C.), one finds the question: "Why is it, that when the Sun passes through quadrilaterals, as for instance in wickerwork, it does not produce a figure rectangular in shape but circular?" Subsequently, scholars in the Middle Ages repeatedly

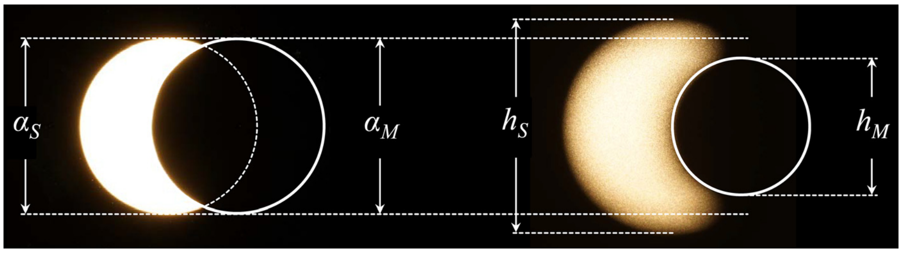


Fig. 1. The Moon puzzle: In direct view (left), Sun and Moon appear to be about the same size: $\alpha_S = \alpha_M$. In contrast, in the pinhole image h_S (right, mirrored for better comparison), the Sun appears enlarged, while the diameter of the Moon h_M, derived from the radius of the obscured part of the Sun, is reduced (Ref. 7).

grappled with this question, albeit without success. Lindberg traces the history of the pinhole camera theory up to the Middle Ages, detailing contributions by natural philosophers such as Al-Kindi, Witelo, Pecham, and others.[8–11] Their explorations reveal intriguing concepts, heuristic considerations, and flawed conclusions, which can be challenging to understand from today's perspective. However, understanding early theories is facilitated by recognizing two distinct cases: point-like and extended apertures, which are often discussed separately by even the same author in different works (Fig. 2).

For a point-like aperture, the camera obscura model was clear, as the rectilinear propagation of light was unquestioned, producing a point-symmetrically mirrored image of the source. For larger apertures, the shape of the aperture becomes crucial, leading to questions about why a square aperture creates a round image, or why the image shape behind the aperture first matches it and then shifts to resemble the luminous object as the image distance increases. These questions challenged the assumption of linear light travel and led to introduction of seemingly peculiar concepts such as primary and secondary rays or an "attenuation" of peripheral light rays.

By the late 16th century, the pinhole camera, likely due to the work of Gemma Frisius,[12] had evolved into an astronomical instrument.[13,14] In this way, the Sun and its eclipses could be comfortably observed in a safe manner. Tycho Brahe also adapted most of his instruments for this purpose. At that time in astronomy, the study of eclipses was of central importance. They provided the only means to determine meridian differences (i.e., longitudes) between different locations and were crucial for studying the relative movements of the Sun and the Moon. Astronomers like Kepler, who supported the Copernican heliocentric model of the solar system, needed to rely on precise eclipse theories to correctly interpret the movements of the planets based on the known motions of the Earth and the Sun. Therefore, there was great interest in deducing correct angular sizes from pinhole images of Sun and Moon.

Many astronomers assumed that they could base their calculations of the Sun's size on the ideal pinhole camera model, where the aperture is treated as point-like. However, discrepancies between the angular size of the Sun observed using other methods and the size calculated from the pinhole image prompted Brahe to refine his calculations by subtracting the pinhole's diameter D from the measured image diameter h_S,

$$h_{S,i} = h_S - D. \tag{1}$$

This approach was empirically effective but lacked theoretical foundation.[15] In a letter to Michael Mästlin,[16] Brahe recounted his observations of the solar eclipse on March 7, 1598. These observations subtly indicated some inconsistencies within the field of astronomy:[17,18]

> Truly it must be acknowledged, that the Moon during a solar eclipse does not appear to be the same size as it appears at other times during full Moons when it is equally far away; but it appears as if it were constricted by about 1/5th, by causes to be disclosed elsewhere. As a result, it appears that the Moon can never obscure the Sun completely, and even if the Moon interposes itself centrally, the remaining light of the Sun encircles it …Although the diameter of the Moon then by our calculations ought to have been $34\frac{3'}{4}$, it could not have appeared in front of the Sun to be more than $28'$, which constriction I recognized and was noticed by no one before me. But experience has taught me thus in observations of the Sun on several occasions when it is eclipsed either in the upper or lower part.

Brahe claimed that the Moon appears about one-fifth smaller during conjunctions (solar eclipses) than during oppositions (full Moons). For Kepler, this observation was remarkable because it suggested that the apparent size of the Moon *changes* even though, according to the prevailing theory, its distance to Earth should remain constant.[19]

Instead of correctly adjusting the Moon's diameter h_M by *adding* the aperture diameter D, Brahe either took the image

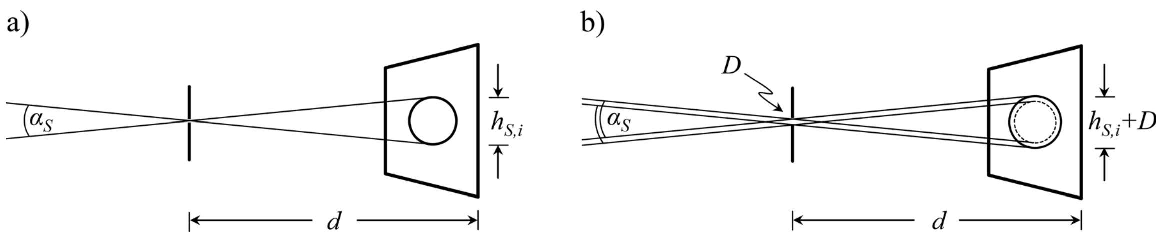
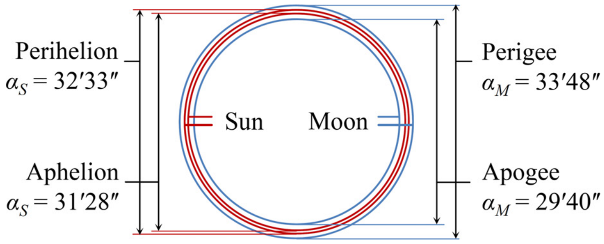


Fig. 2. (a) Relationship between image size $h_{S,i}$, image distance d, and solar angle α_S for ideal pinhole imaging, i.e., expansion-free aperture. (b) If the influence of aperture D is taken into account, the solar image is enlarged.



March 7th, 1598	July 10th, 1600
Wandsbek/Germany	Graz/Austria
53.578° N, 10.087° O	47.068° N, 15.442° O
GMT+00:53:28	GMT+01:05:21
partial eclipse (85,0%)	partial eclipse (48.7%)
Maximum: 11:29	Maximum: 13:39
Sun: 32′11″	Sun: 31′29″
Moon: 32′54″	Moon: 32′10″

Fig. 3. Left: Periodic variations in the solar and lunar angles due to the eccentricity of the Earth's and Moon's orbits. *Perihelion* (January) and *Aphelion* (July) are the closest and furthest points of the Earth's orbit around the Sun. *Perigee* and *Apogee* are the closest and furthest points of the Moon's orbit around the Earth. Right: Reconstructed data on the solar eclipses observed by Brahe 1598 and Kepler 1600 (Ref. 20).

width h_M as the true size or compounded the error by applying the Sun's image correction (Eq. (1)) to the Moon and subtracting the aperture diameter ($h_M - D$), which led to significantly underestimating values for the Moon's apparent size.

To gauge the accuracy of Brahe's observations and astronomical measurements of that period, we compare his values with reconstructed data on the apparent sizes of the Sun and the Moon (Fig. 3).[20] During the "Great Eclipse" on March 7, 1598, Brahe was in Wandsbek, near Hamburg, Germany. At this time, the perihelion was two months previous, and the solar angular size had decreased accordingly by about $1/3'$ to $32'11''$. The Moon had been at perigee on March 5th and had an angular extent of $32'54''$ at noon on March 7th.

When Brahe noticed the reduction in the Moon's size, he updated his lunar tables with the smaller values and informed other astronomers, including Kepler. Instead of considering the imaging conditions of the pinhole camera, Brahe speculated about astronomical causes for the fluctuations in the Moon's size and suggested that the apparent reduction was due to the intensity of sunlight.[3] In contrast, Kepler interpreted Brahe's results as evidence that the Moon, appearing smaller, might be farther away than previously thought, prompting him to question existing astronomical theories.

Other theories suggested that the Moon had a transparent atmosphere that would glow during a full Moon but become translucent when it passed in front of the Sun. While these ideas may seem unusual today, we must recognize the challenges astronomers faced when interpreting images from pinhole cameras. They sought a comprehensive theory that accounted for instrumental conditions like aperture effects as well as astronomical and optical factors. The relationship between observation and theory reveals, which effects arise from which causes.[21,22] Kepler acknowledged this complexity, playfully referring to eclipses as the "eyes of astronomers,"[23] highlighting the close connection between optics and astronomy. He was not convinced by Brahe's ideas, believing the orbits and sizes of celestial bodies were constant. Kepler hoped that observing the solar eclipse of July 10, 1600 in Graz with a new instrument would help resolve the mystery.

B. The solar eclipse of July 10, 1600, and Kepler's solution of the Moon puzzle

For the July 1600 eclipse, Kepler designed a special instrument (Fig. 4), featuring a nearly 4-meter-long axis.[26] This axis could rotate horizontally around a fixed point (azimuth) and be adjusted vertically (altitude). Discs were attached to the axis at specific intervals, positioned perpendicular to its length. The upper disc (M) had a circular aperture, while the lower disc (S) acted as a viewing screen. Kepler's notebook entries begin with a diagram and data sheet detailing this instrument. Here, we examine his notebook entries more closely; a wonderful summary of this episode can be found in Straker.[3]

On July 7, three days before the eclipse, Kepler used his instrument to measure the solar angular diameter but found the values too large. He adjusted his measurements using Brahe's correction method and checked to see if the screen's texture influenced the visibility of the Sun's image. After discovering a discrepancy in the screen-aperture distance, he recalculated, but the diameter remained large. By July 10, Kepler reassessed the Sun's diameter:[23]

> This is 10370 because nothing perceptible was achieved: The diameter of the ray [image] I used appeared, although the light given at the beginning was faint, through watery clouds, so that it could not be seen on blue paper, on white paper it appeared black with $105\frac{1}{2}$ parts, and the aperture with $16\frac{1}{2}$ parts. Therefore, the remainder is 89 parts. Halved, it is $44\frac{1}{2}$.

Kepler's records indicate that the value of 10 370 represents the distance between the aperture and the screen, corresponding to one rod, an old unit of measurement presumably equivalent to 3793 mm.[27] This means that the uncorrected image of the Sun, projected through the pinhole camera, was about 4 cm in size, while the diameter of the pinhole itself was about 6 mm.

In modern notation, the apparent size is given by the relationship $\alpha_S = 2 \cdot \arctan(h_{S,i}/2d)$, where α_S is the solar angle, $h_{S,i}$ is the corrected diameter of the projected image, and d is the image distance. Because the tangent represents a ratio of lengths, we can use Kepler's original values of $h_{S,i}/2 = 44.5$ units and $d = 10370$ units. This yields $\alpha_S \approx 0.4917° = 29'30''$.

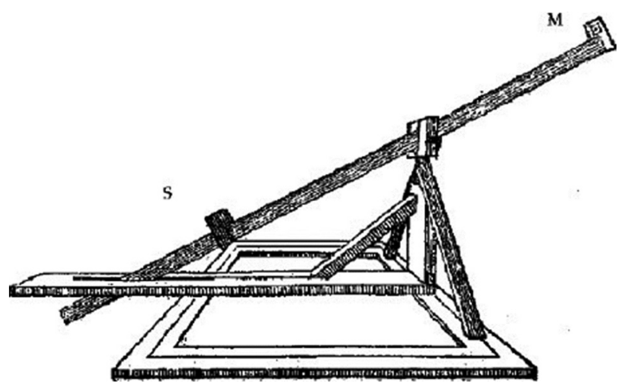


Fig. 4. Kepler's pinhole camera instrument (Ref. 24).

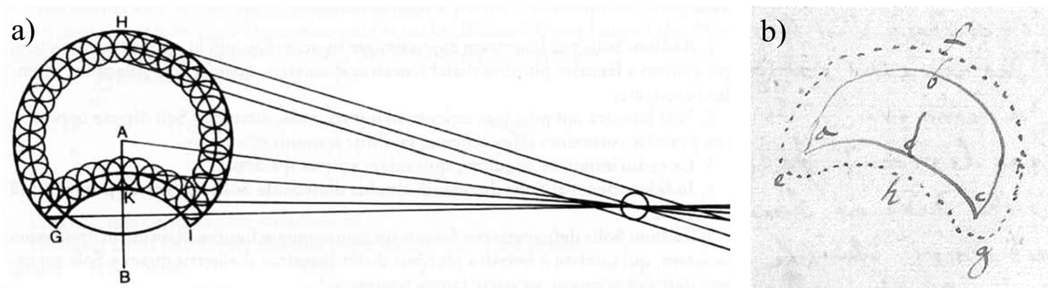


Fig. 5. (a) Kepler's sketch (Ref. 25) from his notebook illustrates the Moon puzzle solution, showing the Moon's diameter reduced and the Sun's enlarged by the aperture's diameter. (b) Kepler's original sketch solving the Moon puzzle, from a letter to Mästlin on September 9, 1600 (Ref. 28).

While Brahe had noted the solar angle as $29'40''$, this 10 arc sec discrepancy prompted Kepler to refine his approach as he proceeded to determine the Moon's diameter:[23]

> I measured the diameter of the Moon only once, and it was not accurate but somewhat smaller than expected. At other times, the image was too faint. From three specially marked points on the lunar circumference and three specially marked points on the solar circumference, the solar radius was clearly determined, i.e., in the correct size, due to the angle of intersection and the optical conditions. The lunar radius was 1 inch and $26\frac{2}{3}$ points or $98\frac{2}{3}$. If $105\frac{1}{3}$ yield $29'30''$, what does $98\frac{2}{3}$ yield?

If the angle of $29'30''$ for the Sun corresponded to $105\frac{1}{2}$ units, then $98\frac{2}{3}$ units would result in a lunar angle of $[(98\ 2/3)/(105\ 1/2)] \cdot 0.4917° = 0.4599°$, which is $27'36''$ for the Moon, consistent with Brahe's false conclusion of around $28'$. However, Kepler continues searching, leading him to reinspect his instrument on July 12th and remeasure the Sun's height. Since thieves had meddled with his equipment, he had inaccurately recorded some length measurements. While Kepler further refined his calculations to pinpoint the error, an unexpected insight surfaced amid his records:[23]

> There occurs to me just now something concerning the diameters of the luminaries [Sun and Moon], why the Moon appears smaller in conjunction than in opposition. The proof emerges clearly from the figure [Fig. 5(a)]. I must only still consider the order of the problems.

Immediately afterward, Kepler developed a novel theory of the camera obscura by systematically formulating theorems and corollaries. He first presents 17 theorems to establish the foundations of a broader theory of the pinhole camera and then applies these to the Moon puzzle in another 14 theorems. From the start, alongside the principle of straight-line light propagation, he introduces a key concept: he views the light source as a collection of an infinite number of point sources. In the case of a point-like light source, a sharp projection of the extended aperture appears on the screen. This image matches the aperture's size when the light source is infinitely far away. Since both the light source and the aperture have finite dimensions, the geometries of both influence the pinhole camera image. The counterpart to the point-like light source is the point-like aperture: in this case, the influence of the aperture on the image is close to zero, resulting in a sharp, point-symmetrically mirrored image of the light source, an ideal pinhole camera. He then specifies a criterion under which the image on the screen tends to match the light source's image: If the distance from the aperture to the screen in multiples of the aperture diameter is not smaller than the distance from the light source to the screen in multiples of the light source diameter, then the shape of the image on the screen deviates from that of the aperture toward that of the light source.

The solar eclipse in Fig. 5(a) exemplifies a special case of Kepler's theory. Central to this is the solution of the old "window problem." A sharp image of the Sun forms only if the aperture is point-like. Now Kepler gradually reveals the solution to the Moon puzzle within his pinhole camera theory, explaining several enigmatic details. For instance, he clarifies why the horns of the solar crescent appear rounded in the pinhole camera image yet sharp in direct view, and why the transition from partial to total eclipse on the pinhole camera screen is abrupt, unlike the gradual change observed in the sky.

Kepler's solution to why the Moon's size appears to shrink during solar eclipses is that the Moon does not actually shrink; rather, the Sun's image, projected through a circular aperture, enlarges and overshadows the Moon's image. For an infinitely distant Sun, the Sun's image expands by the aperture's radius along the entire edge of the solar crescent. Consequently, as the Sun's bright image in the pinhole camera increases by the aperture's diameter, the Moon's obscuring part appears to reduce by the same amount. To calculate the apparent sizes correctly, one must, therefore, adjust the radius of the solar image in the pinhole camera by subtracting the diameter of the aperture from the solar image and adding it to the lunar image (Fig. 6).

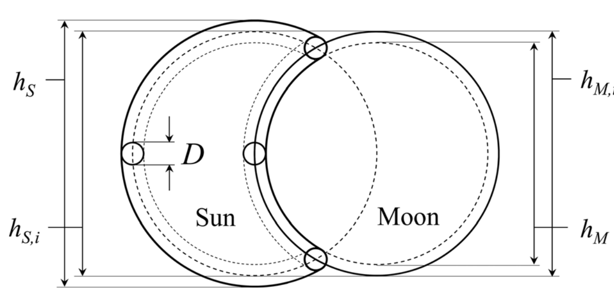


Fig. 6. Kepler's solution to the Moon puzzle: By convolving the image of the partially occulted Sun with the imaging aperture D, the solar image h_S appears enlarged ($h_S = h_{S,i} + D$) and the lunar image h_M reconstructed from the concave edge appears reduced by the same amount ($h_M = h_{M,i} - D$). For simplicity, the size of the images of Sun and Moon is assumed to be the same.

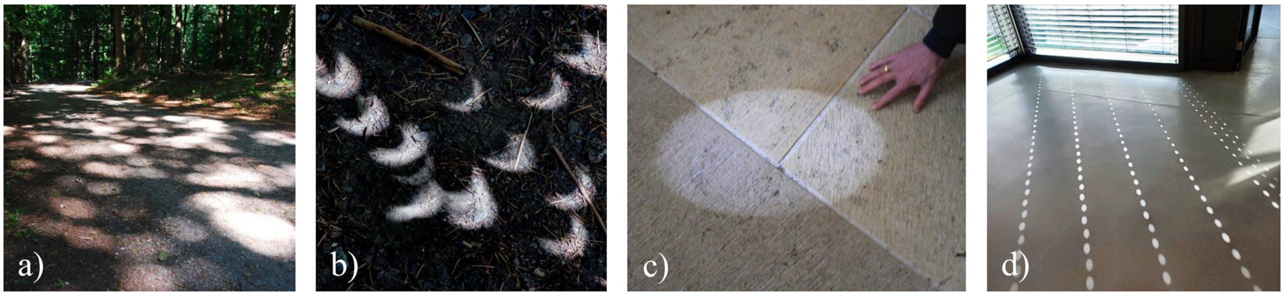


Fig. 7. (a) Solar images on the ground under high leaf canopies are well known and described frequently in the literature (Refs. 33–36). (b) The extent of the influence of size and shape of the solar disc on the appearance of shadows in nature becomes particularly clear during solar eclipses, when the solar disc is partially obscured (Photo: Bill Gozansky). (c) Large solar image on the floor of Regensburg Cathedral and (d) series of solar images as they typically appear at the openings of louvre blinds.

Kepler started by recalculating first by adding the diameter of the aperture to the dark image of the Moon, i.e., $98\frac{2}{3}+16\frac{1}{2}=115\frac{1}{6}$. This gives $\alpha_M \approx 0.6363°$, which is $38'10''$. This value is obviously too large, but it is in the right direction and therefore more in line with his astronomical expectations of a lunar theory wherein the Moon cannot possibly become smaller. To obtain a more realistic value closer to Brahe's reported lunar angle of $34\frac{3}{4}'$, Kepler uses the ratio between the Moon and the Sun, which he states as $11\frac{2}{3}$ to 10. Applied to the solar angle he found, $29'30''$, this ultimately yields a value of $34'25''$ for the Moon.

Kepler not only provided a solution to the Moon puzzle but, in his notes, also presents a generalized theory of the pinhole camera that still holds true today within the framework of geometric optics. This has sometimes been referred to as a milestone in the development of modern optics.[29]

In 1604, Kepler published his optical treatise *Ad Vitellionem Paralipomena*, in which he revealed his theory of the pinhole camera to the scientific community.[25,32] This treatise, particularly in its second chapter, explores the concept of "light figures," including a comprehensive and generalized theory of the pinhole camera. This episode will be described in greater detail in a follow-up paper. For now, we turn to student-level experiments.

III. IMAGING EXPERIMENTS: EXPLORING THE CONTEXT OF THE MOON PUZZLE

The aim of this section is to understand the experimental context of the Moon puzzle using a series of simple observations and qualitative experiments that can be carried out in student laboratories. First, we want to build on everyday observations to develop familiarity with the phenomena and a sensitivity for the relevant quantities and conditions (Fig. 7). Second, we study the imaging conditions of a pinhole and show how the influence of the aperture on the resulting image can be demonstrated using simple equipment. This will provide the experimental basis for understanding Kepler's solution to the Moon puzzle.

A. Experiment I: Creating "Sun coins"

We encounter pinhole images of the Sun, so-called "Sun coins" in everyday life: on the shaded forest floor, on house walls, behind lowered blinds, etc. (Fig. 7). These can easily be produced by crossing the fingers of both hands in front of a sunlit wall (Fig. 8). As one step backward, the shadow image of the finger grid (a) becomes blurred as in (b); the openings in the shadow lose their similarity to the openings in the grid and turn into light circles of the same size: images of the Sun, differing only in brightness and sharpness (c). The smaller the opening in the grid, the darker and also sharper the solar image. If the distance to the wall is increased further, the solar images continue to grow and will overlap.

The mean solar angular size $\alpha_S = 32' \approx 0.53°$ corresponds to a ratio of image size $h_{S,i}$ to image distance d of $h_{S,i}/d \approx 1/108$ rad. If the influence of the aperture is taken into account, the size of the solar images generated with the finger grid can be roughly estimated by using the rule of thumb

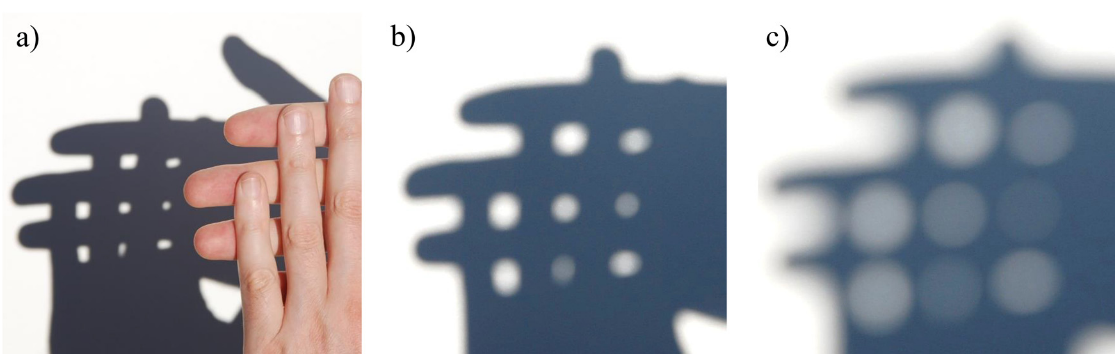


Fig. 8. Sun coins, created in the shadow of crossed fingers on a sunlit wall. The shape of the openings still dominates near the wall (a). As the distance increases (b) and (c), this recedes in favor of the solar images. Photos: Laila Ellis.

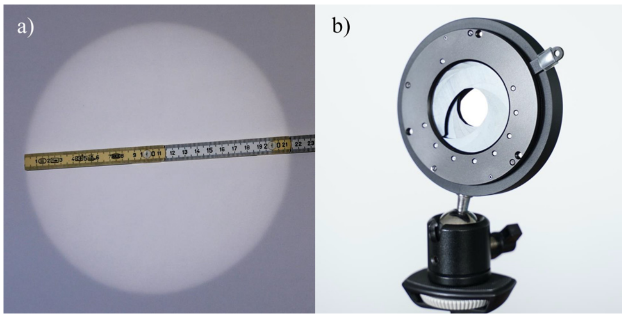


Fig. 9. Pinhole image of the Sun (a), produced with a variable mirror aperture (b).

$h_S/d \approx 1/100$ rad. The size and sharpness of the solar images shown in Fig. 8(c) are not far from those that Kepler is likely to have produced with his open pinhole camera at an image distance of about 4 m.

In order to obtain larger, higher-contrast and sharper images of the Sun, set up a small pocket mirror outside and project the Sun from a greater distance through an open window into a darkened room. For a measurement carried out on October 16, 2017 (calculated deviation of α_S from the mean value of only 6 arc sec), we used a surface mirror ($\varnothing = 40$ mm), whose effective area can be varied with a fully closeable pupil aperture (Fig. 9(b)), which simplifies the positioning of the solar image (large aperture) and, at the same time, allows sharper images (small aperture). With an aperture of $D = 6$ mm and an image distance of $d = 2255.0 \pm 0.5$ cm, we obtained a solar image with a horizontal size of $h_S = 21.4 \pm 0.2$ cm (Fig. 9(a)) and, therefore, get $d/h_S \approx 1/105$. The magnification of the solar image compared to the theoretical value for an expansionless aperture ($h_{S,i} = d/108 \;\; = 20.87$ cm) is therefore $108/105 \approx 3\%$.

To apply Brahe's rule for calculating the size of the solar image, the angle of inclination of the mirror in respect to the image plane must be taken into account. In Fig. 9(a), the horizontal size of the aperture was fully effective. With $h_{S,i} = h_S - D = 20.8$ cm, the deviation of $h_{S,i}/d$ from $1/108$ is less than 0.5%. The seasonal variations of the solar angle between perihelion and aphelion are on the order of 3.3%. The resulting difference in size of the solar image between aphelion and perihelion would be on the order of 0.7 cm in the above case and should be measurable.

B. Experiment II: Simulating and imaging a solar eclipse

To replicate the conditions of a solar eclipse in the laboratory, we use the setup shown in Fig. 10. The light source consists of a hemisphere ($\varnothing = 30$ cm) whose interior is coated with a highly matte white surface and illuminated by four 500 W halogen lamps. A circular aperture ($\varnothing = 19$ cm) made of sheet steel in front of the light source represents the disc of the Sun. The Moon is represented by a cardboard disc glued onto a glass pane, which, in relation to the projection screen, has the *same* angular extent as the light source; we choose this special case for simplicity. Using a pinhole aperture in the projection screen, the scenario is projected onto a second semi-transparent screen, which we observe from behind.

The object distance of the light source from the first screen is $d_1 = 285$ cm, which corresponds to a "solar angle" of $1/15$ rad. With an image distance $d_2 = 45$ cm to the second screen, a "solar image" with a size of about $d_2/15 = 3$ cm can be expected. Figure 11(a) shows the eclipsing phases simulated under the above conditions by moving the lunar disc successively perpendicular to the direction of illumination, as seen from the pinhole (direct view). Figure 11(b) shows the respective pinhole images. The size of the pinhole was $D = 1$ mm, and the solar images were measured to be about $h_S = 31$ mm, as expected.

C. Experiment III: Exploring the influence of the aperture and Brahe's rule

Kepler solved the Moon puzzle by examining the imaging conditions of the pinhole aperture. In order to demonstrate the influence of the pinhole aperture on the size of the image by varying the aperture size, we start with a reproduction of the Moon puzzle. To this end, we compare the direct view of the eclipsing situation with the pinhole image of the partially eclipsed "Sun" for varying aperture size. Figure 12(a) shows a photo of the direct view, scaled to the image size of the ideal pinhole camera image ($h_{S,i} = 30$ mm). Figures 12(b)–12(d) show how the deviation of the values for h_S and h_M from the ideal case increases with larger aperture D.

In Fig. 12(d) with an aperture size of $D = 6$ mm, the pinhole image of the partially eclipsed "Sun" with $h_S = 36 \pm 1$ mm appears significantly enlarged and characteristically rounded at the tips. The lunar diameter reconstructed from the concave edge of the "Sun" is reduced by the same factor: $h_M = 24 \pm 1$ mm. Based on the pinhole image of the

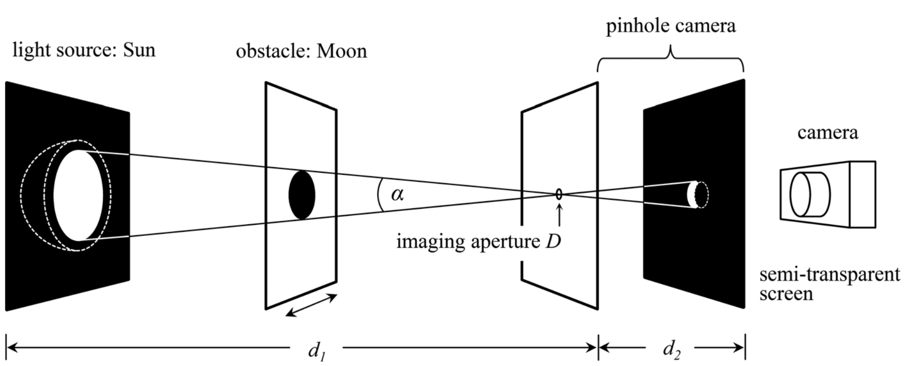


Fig. 10. Setup for replicating and imaging a solar eclipse with a pinhole camera.

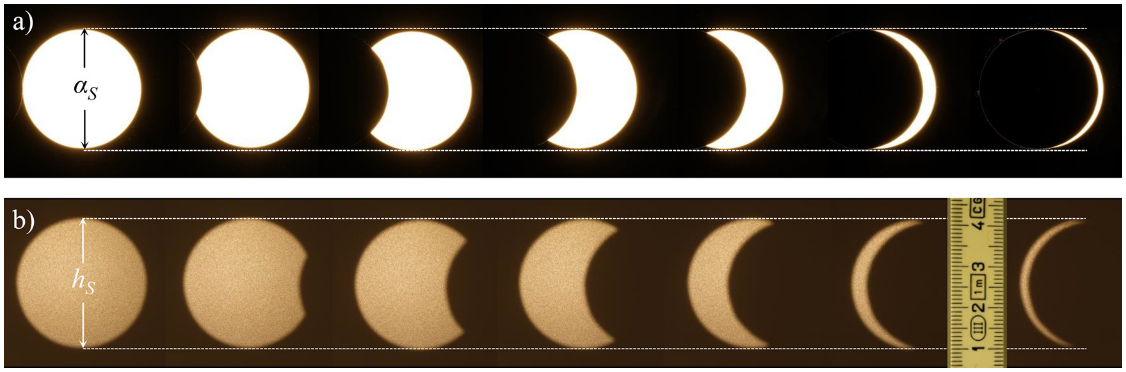


Fig. 11. Replicated solar eclipse in direct view (a) and imaged with the setup from Fig. 10.

eclipsing situation, the following conclusion seems reasonable: As long as the "Sun" is partially eclipsed, the "Moon" appears to be about one-third smaller than at the moment of total eclipse.

Applying Brahe's rule, i.e., subtracting the size of the pinhole D from the resulting solar image size h_S in Table I, results in a good agreement with the calculated value respectively the direct observation. However, we must take into account that the image of the aperture D' generally cannot be equated with the aperture D itself, as is possible in the case of the very distant Sun. With the values given above for d_1 and d_2, the magnification of the aperture image $M = D'/D = (d_1 + d_2)/d_1$ is 15%. For D of Table I, the magnification, thus, remains within the measurement uncertainty of ± 1 mm. With respect to our experimental setup, the assumption $D' \approx D$ seems reasonable for a qualitative reproduction in physics lessons.

The experimental exploration of the aperture's influence on the pinhole image proves that the Moon puzzle is not a consequence of astronomical factors but rather of the imaging properties of the pinhole camera (*optical convolution*). The mystery is solved by realizing what influence the imaging has on what is being imaged.

IV. SUMMARY

We have explored the development of pinhole camera theory following the work of Brahe and Kepler, drawing from original sources. While the question of how "Sun Coins" form was initially of academic interest, the pinhole camera as a standard observational tool highlighted the lack of a comprehensive theory for pinhole imaging, leading to significant difficulties in interpreting astronomical data. Brahe's interpretation of the Moon puzzle challenged several principles of celestial mechanics that Kepler considered certain. Kepler, whose optical insights were crucial for accurate interpretation of astronomical observations, likely faced considerable epistemic pressure, possibly exacerbated by his personal rivalry with Brahe. The historical context presented here can serve as an example of the communication of aspects of the *Nature of Science*[37] or contribute to the field of *Storytelling*.[38,39]

As the cases of Kepler and Brahe illustrate, scientific knowledge is profoundly influenced by historical, cultural, and social factors. Research includes subjective elements, as shown when Brahe interpreted the reduced lunar diameter without clear or comprehensive justification to support his perspective. Furthermore, research is a creative endeavor that does not follow a rigid, step-by-step methodology. This is evident in Kepler's records, which show that the development of his pinhole camera method was not based on a systematic approach but rather on a mix of intuition and serendipity. Therefore, the Moon puzzle is particularly well-suited to being presented as a narrative within the storytelling approach. This strategy allows for the creation of a historical example that accurately reflects the nature of scientific inquiry and discourse.

In the second part of this two-part article (in preparation), we will experimentally and mathematically explore the story of Kepler's "light figures" (soft shadow images). This shows how Kepler's generalizations of his pinhole camera theory became accessible to the scientific community.

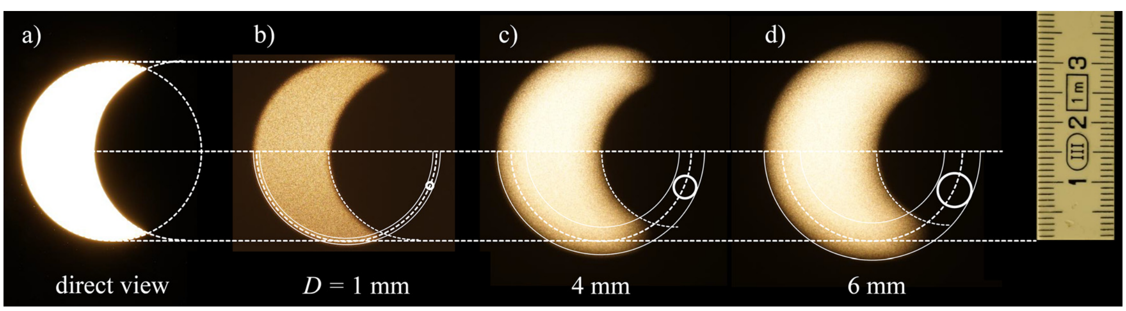


Fig. 12. Influence of the pinhole on the size of the solar image for different pinhole sizes D (b)–(d) compared to the scaled direct view of the partially eclipsed "Sun" (a). Images (b)–(d) have been mirrored to facilitate comparison. The measurement uncertainty of h_S and h_M is on the order of ± 1 mm.

Table I. Deviations of pinhole images of "Sun" h_S and "Moon" h_M during the simulated partial solar eclipse as a function of aperture size D (Fig. 12), given in mm; the measurement uncertainty is ± 1 mm. In agreement with the ideal pinhole scenario, direct observation of the eclipsing situation is set to be $h_S = h_M = 30$ mm.

D	h_S	h_M
1	31	29
4	34	26
6	36	24

ACKNOWLEDGMENTS

The authors thank the reviewers for suggestions, which improved the clarity and presentation of this paper.

AUTHOR DECLARATIONS

Conflict of Interest

The authors declare that they have no conflicts of interest.

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[1]O. Darrigol, *A History of Optics from Greek Antiquity to the Nineteenth Century* (Oxford U. P., 2003).

[2]Ibn Al-Haitham, also known as Alhazen, had already formulated a correct theory of image formation through finite apertures in the 11th century, but his contributions were largely unknown in the western world. His treatise "On the shape of eclipses" remains inaccessible to the West until the 20th century due to its availability solely in Arabic.

[3]S. Straker, "Kepler, Tycho, and the 'Optical Part of Astronomy': The Genesis of Kepler's theory of pinhole image," Arch. Hist. Exact Sci. **24**(4), 267–293 (1981).

[4]V. E. Thoren, *The Lord of Uraniborg. A Biography of Tycho Brahe* (Cambridge U. P., 1990).

[5]J. Marek, "Ansätze zu Youngs und Fresnels Versuchen bei Kepler: Die Rolle der Lochkamera in der Entwicklung der physikalischen Optik," Sudhoffs Arch. **55**(2), 136–151 (1971).

[6]H.-J. Schlichting, "Sonnentaler fallen nicht vom Himmel," MNU **48**(4), 199–204 (1995).

[7]The pictures were taken from the semi-transparent screen in the experimental setup from Fig. 10 with a Sony alpha 7 III digital camera and the 28-75 mm F/2.8 Di III RXD objective from Tamron, as were the pictures in Figs. 9(a), 11, and 12.

[8]D. C. Lindberg, "The theory of Pinhole images from antiquity to the thirteenth century," Arch. Rational. Mech. **5**(2), 154–176 (1968).

[9]D. C. Lindberg, "A reconsideration of Roger Bacon's theory of pinhole images," Arch. Hist. Exact Sci. **6**(3), 214–223 (1970).

[10]D. C. Lindberg, "The theory of pinhole images in the fourteenth century," Arch. Hist. Exact Sci. **6**(4), 299–325 (1970).

[11]D. C. Lindberg, *Theories of Vision From Al-Kindi to Kepler* (University of Chicago Press, 1996).

[12]This limitation likely stems from Flemish mathematician and geographer Gemma Frisius's *De Radio Astronomica et Geometrico* (1544). Frisius, the first to document using the camera obscura to observe a solar eclipse, mistakenly assumed the aperture's size was negligible if sufficiently small. Nevertheless, his work helped establish the camera obscura as a key astronomical instrument.

[13]J. L. Mancha, "Astronomical use of pinhole images in William of Saint-Cloud's Almanach planetarum (1292)," Arch. Hist. Exact Sci. **43**(4), 275–298 (1992).

[14]J. Wlodarczyk, "Solar eclipse observations in the time of Copernicus: Tradition or novelty?," J. Hist. Astron. **38**(3), 351–364 (2007).

[15]Brahe never explicitly formulated or documented his correction method; instead, he applied it implicitly. Nevertheless, he was the first to explicitly record the size of the aperture in his notes. Straker[3] suggests a way in which Brahe might have arrived at a geometric justification without possessing a complete theory of the pinhole camera.

[16]Michael Mästlin (1550–1631), a professor at the University of Tübingen, was one of Kepler's key teachers and mentors and an early open supporter of the Copernican theory. Under Mästlin, Kepler learned about Copernican cosmology and became a staunch advocate of the heliocentric system. He likely also learned about the pinhole camera method from Mästlin. Despite later skepticism toward some of Kepler's ideas, like elliptical orbits, Mästlin remained a vital correspondent for Kepler until his death.

[17]T. Brahe, in *Opera Omnia*, edited by J. L. E. Dreyer (Libraria Gyldendaliana, Hauniae, 1913), Vol. XII, p. 55.

[18]We reproduce the quote from Straker[3] here. Through this letter, which Mästlin forwarded, Kepler likely first became aware of this problem. Before this, he had neither corresponded with Brahe nor met him personally, nor had he observed a solar eclipse.

[19]It is a historical irony that Kepler, likely around 1605, conceived the idea of distance-varying elliptical orbits while analyzing Brahes data.

[20]There are various websites that contain tables with calculated perigee and apogee values going back to the 16th century or allow the calculation of astronomical data at any point in time. The values we use come from <https://www.astropixels.com/ephemeris/moon/moonperap1501.html and https://planetenrechner.de/>.

[21]G. Hon, "On Kepler's awareness of the problem of experimental error," Ann. Sci. **44**(6), 307–345 (1987).

[22]G. Hon and Y. Zik, "Kepler's Optical Part of Astronomy (1604): Introducing the Ecliptic Instrument," Perspect. Sci. **17**(3), 307–345 (2009).

[23]J. Kepler, "Manuscripta astronomica (III)," in *Gesammelte Werke*, edited by V. Bialas, F. Boockmann, W. von Dyck, E. Knobloch, and M. Casper (C.H. Beck, Munich, 2002), Vol. 21.1.

[24]This sketch, found in his later work *Astronomiae Pars Optica* from 1604, is more detailed than the original illustration in Kepler's notebook (Ref. 25).

[25]J. Kepler, "Astronomiae pars optica," in *Gesammelte Werke*, edited by J. Hammer and M. Caspar (C.H. Beck, Munich, 1939), Vol. 2.

[26]C. Sigismondi and F. Fraschetti, "Measurements of the solar diameter in Kepler's time," Observatory **121**, 380–385 (2001).

[27]Prior to the adoption of a standardized metric system, measurements varied widely, with each nation or region adhering to its own unique system. In the Archduchy of Austria, the prevailing unit of length was the Werkrute, a system based on a 12-foot standard. One foot in this system corresponded to 316.081 mm, resulting in a Werkrute (equivalent to 12 ft) measuring 3792.968 mm.

[28]J. Kepler, "Briefe 1599-1603," in *Gesammelte Werke*, edited by M. Hammer and W. von Dyck (C.H. Beck, Munich, 1949), Vol. 14.

[29]Simultaneously, the notion of theories emerging ex nihilo – sudden, isolated bursts of insight – merits critical examination. It is tempting, yet often misleading, to frame scientific progress as a heroic narrative, a "quasi-history."[30,31] Kepler, like any thinker, operated within a vibrant scientific-sociological network. He engaged in lively correspondence on astronomical puzzles, including the Moon puzzle, with figures like astronomer Michael Mästlin and scholar-diplomat Herwart von Hohenburg. His visit to Tycho Brahe in Prague in February 1600 marked the start of a fruitful collaboration, likely exposing Kepler to the correction method for solar pinhole images and potentially a geometric underpinning.

[30]M. A. B. Whitaker, "History and quasi-history in physics education—Part I," Phys. Educ. **14**(2), 108–112 (1979).

[31]M. A. B. Whitaker, "History and quasi-history in physics education—Part 2," Phys. Educ. **14**(4), 239–242 (1979).

[32]J. Kepler, *Optics*, edited by William H. Donahue (Green Lion Press, Santa Fe. New Mexico, 2000).

[33]A. J. Mallmann, "Tree leaf shadows to the Sun's density: A surprising route," Phys. Teach. **51**(1), 10–11 (2013).

[34]P. Hewitt, "Pinhole image of the Sun," Phys. Teach. **38**(5), 272–272 (2000).

[35]T. B. Greenslade, Jr., "Pinhole images of the eclipsing Sun," Phys. Teach. **32**(6), 347–347 (1994).

[36]V. Kriss, "Measuring pinhole images of the Sun," Phys. Teach. **34**(3), 190–191 (1996).

[37]W. F. McComas, *The Nature of Science in Science Education: Rationales and Strategies* (Springer, Dordrecht, 2002).

[38]Y. Hadzigeorgiou, "Narrative thinking and storytelling in science education," in *Imaginative Science Education* (Springer, 2016).

[39]P. Heering, "False friends: What makes a story inadequate for science teaching?," Interchange **41**(4), 323–333 (2010).